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UNSTEADY MOTION OF A THIN AIRFOIL UNDER THE FREE SURFACE OF A WEIGHTLESS FLUID OF FINITE DEPTH

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The problem of unsteady motions of a thin airfoil under the free surface /115*
of a weightless fluid of finite depth is discussed in the linear formulation. The analogy to flow around a cascade with an infinite number of special-shape airfoils is employed. The method of discrete vortices is used to solve the latter problem.

Let us consider small harmonic oscillations of an infinitely thin airfoil of length 2 c in a plane-parallel flow of a weightless incompressible fluid with constant undisturbed velocity V_0, bounded by a free surface above and by a solid bottom below.

The origin of the body axis system will be taken at the center of some mean position of the oscillating airfoil. The Ox axis will be directed against the incident flow, and the Oy axis, vertically upward. The mean submersion depth of the airfoil under the free surface will be denoted by h, and the mean distance from the bottom, by h_0. In the case of harmonic oscillations of frequency p, the disturbance potential may be represented as

$$\phi(x, y, t) = \overline{\phi}(x, y)e^{ipt}.$$

Here and below, only the real part of the expressions containing the time factor in exponential form should be used.

For the reduced potential $\overline{\phi}(x, y)$, a linear boundary value problem will be obtained on the basis of the accepted assumptions.

It is necessary to find the solution of Laplace's equation

$$\Delta\overline{\phi} = 0, \qquad (1)$$

satisfying the boundary conditions

$$\overline{\phi}_x = 0 \text{ for } y = h \quad (2); \qquad \overline{\phi}_y = 0 \text{ for } y = -h_0; \qquad (3)$$

$$\overline{\phi}_y = V_y(x) \text{ for } y = 0; \qquad |x| \leqslant c; \qquad (4)$$

$$\overline{\phi}_x \to 0; \quad \overline{\phi}_y \to 0 \text{ for } x \to \infty. \qquad (5)$$

In addition, according to the requirement of the Zhukovskiy-Chaplygin hypothesis, the velocity boundedness condition in the trailing edge of the airfoil should be fulfilled for $y = 0$ and $\overline{x} = -c$.

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*Numbers in the right margin indicate pagination in the original text.

The boundary value problem (1)-(5) will be solved by the method of singularities, using the layer of bound vortices along a segment of the Ox axis ($y = 0$; $-c \leqslant x \leqslant c$). The strength of the free vortices of the airfoil and /116
wake will be determined on the basis of the Birnbaum-Küssner unsteady flow pattern.[2]

It is easy to show that the problem under consideration is reduced to the problem of unsteady flow past some special-type infinite lattice. Such an analogy is based on the fact that condition (2) on a free surface in vortex flow can be satisfied by placing a vortex with a strength of the same magnitude and sign at a distance h above the free surface.

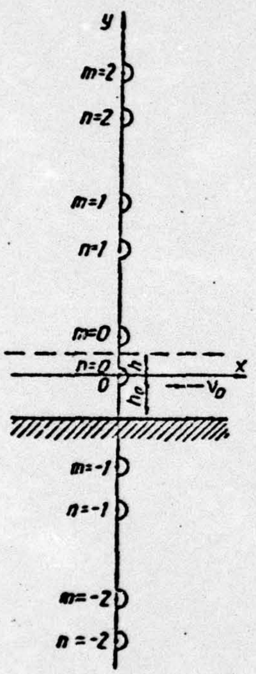

FIGURE 1

To fulfill the condition on the solid boundary (3), a vortex with a strength of equal magnitude but opposite sign must be positioned symmetrically relative to this boundary. Thus, for a single bound vortex in a flow of fluid of finite depth, we arrive at the vortex chain shown in Fig. 1.

We will determine the vertical velocity, caused by the vortex chain in question, at some point of the flow M(x, y). If the vortex strength is constant,

$$v_{y0}(x, y) = \frac{\Gamma_0}{2\pi}\left\{\sum_{-\infty}^{+\infty}\frac{(-1)^n x}{x^2 + [y - 2n(h + h_0)]^2} + \sum_{-\infty}^{+\infty}\frac{(-1)^m x}{x^2 + [y - 2h - 2m(h + h_0)]^2}\right\}.$$

At the points of the Ox axis, we have

$$v_{0y}(x, 0) = \frac{\Gamma_0}{2\pi}\left\{\frac{1}{x} + 2\sum_{1}^{\infty}\frac{(-1)^n x}{x^2 + 4[m(h + h_0) + h]^2} + \sum_{-\infty}^{+\infty}\frac{(-1)^m x}{x^2 + 4[m(h + h_0) + h]^2}\right\}. \tag{6}$$

Using the equality

$$\frac{\pi}{\operatorname{sh}\pi z} = \frac{1}{z} + \sum_{k=1}^{+\infty}\frac{(-1)^k z}{z^2 + k^2},$$

we sum up expression (6) and obtain

$$v_{0y}(x) = \frac{\Gamma_0}{2\pi}\operatorname{csch}\frac{\pi x}{2(h + h_0)} \times \left[1 + \frac{\cos\dfrac{\pi h}{h + h_0}}{\cos^2\dfrac{h}{h + h_0}\pi + \sin^2\dfrac{h}{h + h_0}\pi\,\operatorname{cth}^2\dfrac{\pi x}{2(h + h_0)}}\right]. \tag{7}$$

For a variable vortex strength $\Gamma = \Gamma(t)$, the velocities in the plane of flow are induced not only by point vortices, but also by their vortex sheets with a linear vortex strength /117

$$\varepsilon(x, t) = -\frac{1}{V_0}\left[\frac{d\Gamma(\tau)}{d\tau}\right]_{\tau = t - \frac{x}{V_0}}. \tag{8}$$

Let

$$\Gamma(t) = \Gamma_0 e^{ipt}.$$

Then on the basis of the Birnbaum-Küssner hypothesis, the vertical velocity is given by

$$v_y(x) = (v_y^{(1)} + i v_y^{(2)})\, e^{ipt}, \tag{9}$$

where

$$\begin{aligned} v_y^{(1)} &= v_{y0}(x) - k \int_{-\infty}^{0} \sin ks' v_{y0}(x - s')\, ds'; \\ v_y^{(2)} &= -k \int_{-\infty}^{0} \cos ks' v_{y0}(x - s')\, ds'; \end{aligned} \tag{10}$$

$k = \frac{pc}{V_0}$ is the Strouhal number.

To find the nonsteady aerodynamic characteristics of such an airfoil, it is necessary to determine the strength of the bound vortices of the airfoil. On the basis of the no-flow condition (4) of a continuous vortex layer, expressions (9) and (10) make it possible to write an integral equation for finding the unknown strength.

The nonsteady aerodynamic characteristics of the airfoil are conveniently expressed in terms of the coefficients of rotary derivatives.[1]

The unknown vortex strength of a nondeformable airfoil will be represented in the linear approximation as follows:

$$\gamma = \alpha\gamma_1^{(1)} + \frac{c}{V_0}\dot{\alpha}\gamma_1^{(2)} + \frac{c}{V_0}\omega\gamma_2^{(1)} + \frac{c^2}{V_0^2}\dot{\omega}\gamma_2^{(2)}, \tag{11}$$

where α is the angle of attack; ω is the absolute angular velocity of the body axis system, $\dot{\alpha} = \frac{d\alpha}{dt}$; $\dot{\omega} = \frac{d\omega}{dt}$; the coefficients $\gamma_1^{(2)}, \ldots, \gamma_2^{(2)}$ are referred to as the rotary derivatives of the vortex strength.

The kinematic characteristics of the airfoil will be assumed to change in accordance with the laws

$$\alpha = A_1 e^{ipt}; \qquad \omega = \frac{V_0}{c} A_2 e^{ipt}. \tag{12}$$

In view of expressions (12), the vortex strength (11) will be written as follows:

$$\gamma = \sum_{\nu=1}^{2} A_\nu [\gamma_\nu^{(1)} + ik\gamma_\nu^{(2)}]\, e^{ipt}. \tag{13}$$

We transform the no-flow conditions

$$v_y = -V_0\alpha + \omega x$$

by means of formulas (12) to the form

$$\frac{v_y}{V_0} = \left[-A_1 + \frac{x}{c}A_2\right]e^{i\varphi}. \tag{14}$$

To set up the integral equation, it is necessary to calculate the vertical velocity caused by the adjacent surface layer with strength (13), and subordinate it to the condition (14). The equation thus obtained turns out to be singular and in addition, to contain a logarithmic type singularity - (cikx). /118

Separating the real from the imaginary parts in the integral equation, and in the imaginary parts, the terms containing equal coefficients A_1 or A_2, we obtain two systems of integral equations, which we write in dimensionless form

$$\begin{aligned} &\int_{-1}^{+1}\gamma_1^{(1)}(\bar{s})\,v_y^{(1)}(\bar{x}-\bar{s})\,ds - k\int_{-1}^{+1}\gamma_1^{(2)}(\bar{s})\,v_y^{(2)}(\bar{x}-\bar{s})\,d\bar{s} = -2\pi;\\ &\int_{-1}^{+1}\gamma_1^{(1)}(\bar{s})\,v_y^{(2)}(\bar{x}-\bar{s})\,ds + k\int_{-1}^{+1}\gamma_2^{(2)}v_y^{(1)}(\bar{x}-\bar{s})\,d\bar{s} = 0; \end{aligned} \tag{15}$$

$$\begin{aligned} &\int_{-1}^{+1}\gamma_2^{(1)}(\bar{s})\,v_y^{(1)}(\bar{x}-\bar{s})\,d\bar{s} - k\int_{-1}^{+1}\gamma_2^{(2)}(\bar{s})\,v_y^{(2)}(\bar{x}-\bar{s})\,d\bar{s} = 2\pi\bar{x};\\ &\int_{-1}^{+1}\gamma_2^{(1)}(\bar{s})\,v_y^{(2)}(\bar{x}-\bar{s})\,d\bar{s} + k\int_{-1}^{+1}\gamma_2^{(2)}(\bar{s})\,v_y^{(1)}(\bar{x}-\bar{s})\,ds = 0. \end{aligned} \tag{16}$$

Here

$$\bar{x} = \frac{x}{c}; \quad \bar{s} = \frac{s}{c}; \quad \bar{\gamma} = \frac{\gamma}{V_0}.$$

The method of discrete vortices (1) will be used to solve the system of integral Equations (15) and (16).

Mathematically, this method signifies the replacement of definite integrals within the limits (-1, +1) by formulas of mechanical quadratures with nodes at points $s_j = -1 + \frac{4j - 1}{4N}$ and an approximate fulfillment of the equations of the system at some points $x_i = -1 + \frac{4i - 3}{4N}$ close to nodes s_j. Such a shift of variables in singular integrals of the form

$$\int_{-1}^{+1}\gamma(s)\frac{M(x,s)}{x-s}\,ds, \qquad -1 \leqslant x < 1$$

makes it possible to avoid kernel singularities when replacing an integral equation by a system of algebraic ones.

In the indicated arrangement of points x_i and s_j, a boundedness of the function $\gamma(x)$ is reached when $x \to -1$; when $x \to 1$, it has a singularity of integrable order. For other classes of functions $\gamma(x)$, other layouts of points x_i and s_j can be proposed.

The aerodynamic coefficients will be represented in terms of the total rotary derivatives

$$C_y = \alpha C_y^{\alpha} + \frac{c}{V_0}\dot{\alpha} C_y^{\dot{\alpha}} + \frac{c\omega}{V_0} C_y^{\omega} + \frac{c^2\dot{\omega}}{V_0^2} C_y^{\dot{\omega}};$$

$$C_m = \alpha C_m^{\alpha} + \frac{c}{V_0}\dot{\alpha} C_m^{\dot{\alpha}} + \frac{c}{V_0}\omega C_y^{\omega} + \frac{c^2}{V_0^2}\dot{\omega} C_y^{\dot{\omega}},$$

which, according to the formulas /119

$$C_y = \int_{-1}^{+1} \gamma(\bar{x})\, d\bar{x}; \qquad C_m = \int_{-1}^{+1} \bar{x}\gamma(\bar{x})\, d\bar{x},$$

are expressed in terms of the distributed rotary derivatives $\gamma_1^{(1)}, \ldots, \gamma_2^{(2)}$.

When the method of discrete vortices is used, the integrals will be replaced by the finite sums

$$C_y^{\alpha} = \frac{2}{N}\sum_{j=1}^{N} \gamma_{1j}^{(1)}; \qquad C_m^{\alpha} = \frac{2}{N}\sum_{j=1}^{N} s_j \gamma_{1j}^{(1)};$$

$$C_y^{\dot{\alpha}} = \frac{2}{N}\sum_{j=1}^{N} \gamma_{1j}^{(2)}; \qquad C_m^{\dot{\alpha}} = \frac{2}{N}\sum_{j=1}^{N} s_j \gamma_{1j}^{(2)};$$

$$C_y^{\omega} = \frac{2}{N}\sum_{j=1}^{N} \gamma_{2j}^{(1)}; \qquad C_m^{\omega} = \frac{2}{N}\sum_{j=1}^{N} s_j \gamma_{2j}^{(1)};$$

$$C_y^{\dot{\omega}} = \frac{2}{N}\sum_{j=1}^{N} \gamma_{2j}^{(2)}; \qquad C_m^{\dot{\omega}} = \frac{2}{N}\sum_{j=1}^{N} s_j \gamma_{2j}^{(2)},$$

where N is the number of bound vortices.

The validity criterion of the calculations is the requirement $C_y^{\omega} = C_m^{\alpha}$ and $C_y^{\omega} = C_m^{\alpha}$, which is a corollary of the reversibility theorem.[1]

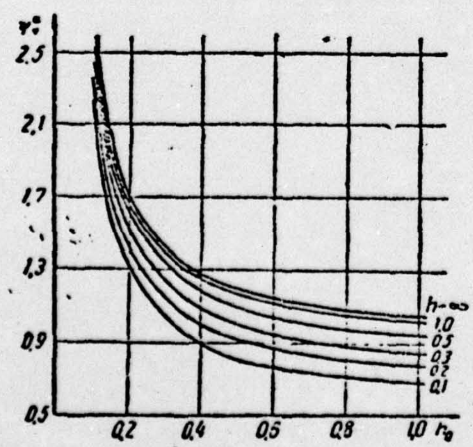

FIGURE 2

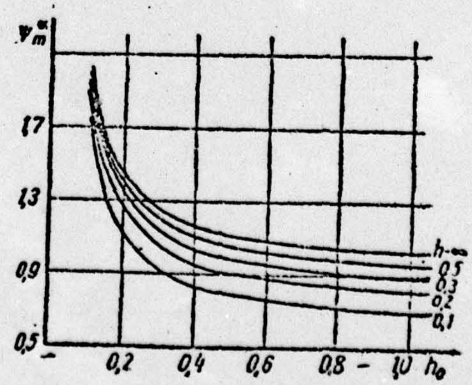

FIGURE 3

Figures 2 and 3 are plots of the change in the functions $\psi_y^\alpha = \frac{C_{yhh_0}}{C_\infty^\alpha}$ and $\psi_m^\alpha = \frac{C_{mhh_0}^\alpha}{C_{m\infty}^\alpha}$ in steady motion (k = 0) with the relative distance of the thin airfoil from the bottom. It follows from these figures that as the distance increases, the aerodynamic coefficients of the airfoil approach the values of the corresponding characteristics during motion under the free surface of a fluid of infinite depth. As the distance of the airfoil from the bottom decreases, its aerodynamic characteristics increase and approach the corresponding values for motion near a solid boundary. /120

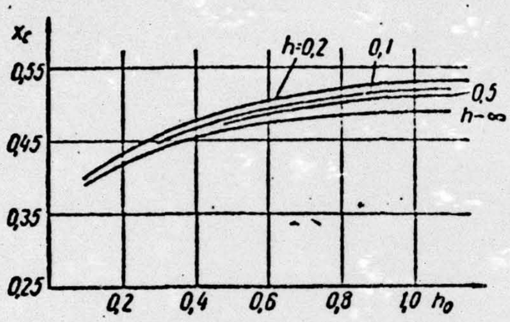

FIGURE 4

Figure 4 shows the change in the position of the pressure center of a thin airfoil, a change barely noticeable near the free surface. As the solid boundary is approached, the pressure center shifts appreciably toward the midsection of the airfoil.

The numerical results obtained for steady flow at relative submersions and distances $h > 0.25$, $h_0 > 0.25$ are in good agreement with the results of A. N. Panchenkov's calculations,[3] obtained by using the asymptotic small-parameter method. At small submersions and distances, the method described makes it possible to obtain results corresponding more accurately to the initial equation.

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